A survey of recent empirical work concerning auctions

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Abstract. In this paper, we survey some recent empirical work concerning auctions, first outlining two complementary approaches to the empirical analysis of auctions, and then discussing several recent developments in the econometric analysis of field data concerning auctions.

Une revue des travaux empiriques récents sur les enchères. Dans ce mémoire, les auteurs passent en revue certains travaux empiriques récents sur les enchères. D'abord, ils esquissent deux approches complémentaires qui ont été utilisées dans l'analyse empirique des enchères. Ensuite, ils discutent plusieurs développements récents dans l'analyse économétrique des données collectées sur le terrain à propos des enchères.

I. INTRODUCTION

The development of appropriate game-theoretic tools has made the study of auctions one of the most fruitful areas of theoretical research during the past decade; see Milgrom (1985, 1987) and McAfee and McMillan (1987) for surveys. Auctions are of special interest to economists because they are explicit mechanisms that describe how prices are formed. In addition, many transactions (particularly between the public and private sectors) involve auctions, so understanding how they work has practical value. Recently, a number of empirical researchers have used game-theoretic models of auctions to interpret actual field data from these institutions. In this paper, we discuss this research, focusing upon the problems encountered in taking the theory to data and the merits of proposed solutions.

The main components of an auction model consist of a set of potential buyers, the joint distribution of valuations for these potential buyers, and a reserve price rule

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used by the seller. Each potential buyer is assumed to know his valuation, but not those of his opponents. The probability law determining the valuations of potential buyers is assumed to be common knowledge. The auction rules determine what is bid and the pay-off to each potential buyer as a function of submitted bids. When formulated as a game of incomplete information, the Bayesian-Nash equilibrium of the model expresses each potential buyer's bid as a function of his valuation, the number of potential buyers, the reserve price, and the distribution of valuations of opponents conditional upon his own valuation.[1] This in turn can be used to determine the distribution of the winning bid, the market price.

Recent empirical research has had two main goals: The first is to test the behavioural theory. Do potential buyers bid according to a Bayesian-Nash equilibrium? If the valuations of potential buyers and the probability law determining these valuations are known to the researcher, then this question is easily addressed by comparing the bids submitted with those predicted by the equilibrium bidding strategy. This approach is followed in experiments where researchers choose the probability law and draw the valuations for potential buyers. See, for example, the work of Cox, Robertson, and Smith (1982); Cox, Smith, and Walker (1985, 1988); Dyer, Kagel, and Levin (1989); and Kagel and Levin (1986). Experimental studies suffer from the problem that the behaviour of subjects in an experiment may differ from that of agents who participate in real-world markets. The experience of the experimental subjects may be too limited and the compensation too small for them to perform the requisite optimizing calculations. Also, the subjects in experiments are usually drawn randomly from a population of university students, whereas agents in a real-world market are typically a select group, survivors of a process in which those who do not perform well are not given decision-making power. (See Harrison 1989 for further discussion on these points.) Consequently, while informative, experimental studies are not a substitute for the careful analysis of field data. The difficulty with field data, on the other hand, is that neither the valuations of potential buyers nor the probability law determining these valuations is observed by the researcher.

The second goal of recent empirical research is to identify the probability law governing the valuations of potential buyers. This exercise is central to implementing optimal selling mechanisms. The literature concerning mechanism design has been criticized as lacking practical value because the optimal mechanism depends upon random variables whose distributions are typically unknown to the designer. In auctions, the equilibrium bid strategy of a potential buyer is usually an increasing function of his valuation. Consequently, if the researcher assumes that potential buyers bid according to Bayesian-Nash equilibrium strategies, then it may be possible to estimate the underlying probability law of valuations using bid data from a cross-section of auctions. The estimated distribution can be used to determine the revenue-maximizing selling mechanism. For example, Paarsch (1993)

1 If the reserve price and/or the number of potential buyers are unknown to a potential buyer at the time he chooses his bid, then the prior distributions of these variables conditional upon his valuation are what matter. An application where the reserve price and the number of potential bidders are random is the auctioning of oil and gas leases on offshore U.S. federal lands.

estimated the optimal auction to sell timber cutting rights in British Columbia, Canada, after estimating the probability law of valuations from bid information from oral ascending-price (English) auctions.

Hendricks and Porter, with a number of co-authors, have pursued the first goal using data from auctions of drainage leases on offshore U.S. federal lands. These are leases which are adjacent to tracts upon which an oil or a gas deposit (or both) has been discovered. The evidence suggests that firms owning the adjacent tracts, called 'neighbour firms,' are well informed concerning the value of the drainage lease and coordinate their bidding decisions. Non-neighbour firms are relatively uninformed. Under these conditions, the game-theoretic model of bidding provides a number of testable predictions about the relative profits of neighbour and non-neighbour firms, and their bidding behaviour. These restrictions do not depend upon the functional form of the probability law for valuations. Furthermore, in the absence of unobserved heterogeneity across auctions, knowledge of the valuations or the probability law determining these valuations is unnecessary to test formally the implied restrictions.

Paarsch (1989, 1991, 1992, 1993); Laffont, Ossard, and Vuong (1991); and Elyakime et al. (1994) have been primarily interested in the second goal, identifying the exact data-generating process. The theory is assumed to hold and the equilibrium behavioural relationships are imposed upon the data to determine the underlying probability law. This approach is known among econometricians as structural estimation. It admits observed covariate heterogeneity in a straightforward and model consistent way, and can admit some forms of unobserved heterogeneity. The main difficulty with the structural approach is the complexity of the equilibrium bid functions, which are often highly non-linear and, in some cases, do not have closed-form representations.

The estimation procedures developed by Donald and Paarsch (1991, 1993a) require that the joint distributions belong to particular families of distributions that admit closed-form solutions to the bid functions that can be solved using numerical methods. Empirical specifications can then be estimated using non-linear programming techniques to obtain the maximum likelihood estimator. Laffont, Ossard, and Vuong (1991) have proposed a simulated non-linear least squares estimator that allows for a larger class of distribution functions. Nevertheless, a specific distributional assumption must still be made. More recently, Elyakime et al. (1994) have implemented the non-parametric methods of Guerre et al. (in progress) to estimate the probability law of valuations. This method essentially imposes no restrictions upon the class of admissible probability laws.

Structural estimation can also provide tests of behavioural theory. Paarsch (1989, 1991) used data from first-price sealed-bid and oral ascending-price (English) auctions of timber cutting rights to compare the estimated probability laws for valuations. If the choice of mechanism is random (which appears to be the case) and the theory holds, then the estimated distributions should be identical. The non-parametric estimator of Guerre et al. as implemented by Elyakime et al. (1994) does not require that the bid function be increasing or equal to the reserve price when evaluated at that price. Consequently, if the estimated bid function has these properties, this is evidence supporting the behavioural hypothesis. Donald and Paarsch (1993b) illustrate a variety of specification tests which can be applied to test the structural models.

All of the papers on structural estimation that we shall be surveying in this paper are within the independent private values paradigm.[2] Potential buyers are assumed to be risk neutral with respect to winning the object, and their valuations are assumed to be independent draws from a common probability distribution. This is the simplest possible bidding environment within which to identify the probability law determining buyer valuations. The assumption of independently and identically distributed draws implies that the joint distribution of buyer valuations factors into N identical marginal distributions, where N is the number of potential buyers. This substantially reduces the dimensionality of the estimation problem. Extensions of the econometric model to allow for risk preferences (see Paarsch 1989, 1991) and affiliation of buyer valuations are subjects of ongoing research.

This paper is organized as follows. In section II, we discuss the work of Hendricks and Porter on drainage auctions. In section III, we introduce the reader to structural econometric models of English auctions. In section IV, we discuss the work of Paarsch and Donald, and of Laffont and Vuong and co-authors concerning first-price sealed-bid auctions. In section V, we summarize the paper and conclude.

II. NON-STRUCTURAL APPROACH

In this section, we first introduce the theoretical model that Hendricks and Porter (with co-authors) developed and used to analyse drainage auctions. We then discuss the key empirical predictions of the model and the approach which Hendricks and Porter adopted for testing these predictions.

Consider the following bidding environment. An indivisible object with unknown common value V is to be sold at a first-price sealed-bid auction. The participants consist of an informed seller who observes a private signal and sets a positive reserve price R before the bids are revealed, an informed buyer who observes a private signal X, and an arbitrary number of uninformed buyers who only observe a public signal Z. The signals lie in a P-dimensional space. The joint distribution of (V, X, R) given $Z = z$ is common knowledge and denoted by $F(v, x, r; z)$. Potential buyers submit sealed bids without knowing the reserve price. A bidder wins the object with certainty if his bid is strictly greater than any other bid and not less than the reserve price. If more than one bidder submits the same bid and those bids are greater than the reserve price, then the object is allocated according to some rationing rule that is independent of the reserve price. The winner of the auction receives the object in exchange for his bid.

This bidding environment is a good description of the sale of drainage leases by the U.S. government off the coasts of Louisiana and Texas. There V represents

2 Paarsch (1992) has considered simple structural models within the other main theoretical paradigm, the common value paradigm.

the value of the unknown amounts of oil and gas that can be extracted from the drainage tract; X represents geological information that neighbour firms obtain from drilling wells on their leases; and Z represents the information which non-neighbour firms possess, seismic information and observable production on the adjacent tracts. Working with data from tracts sold during the period 1959 to 1979, Hendricks and Porter (1988, 1993) established that neighbour firms were considerably better informed concerning the value of the drainage tracts than non-neighbour firms. They also provided evidence of collusion among the neighbour firms, suggesting that effectively only one informed potential buyer existed.

The pre-announced minimum (reserve) price for drainage leases was \$25 per acre. Most tracts were 2,250 acres, which implies that any bid less than \$67,250 was rejected. However, the government reserved the right to reject higher bids if it believed those bids were too low. The basis upon which the government made this judgment was its own private estimate of the value of the lease (obtained from the U.S. Geological Survey) and the number of actual bidders. Thus, from the perspective of the potential buyers the reserve price was effectively a random variable that was likely to be correlated with (V, X). The highest bid was rejected on approximately 20 per cent of the leases. Although the rejection decision was made after the bids were submitted, Hendricks, Porter, and Spady (1989) found no evidence of strategic behaviour upon the part of the government. The rejections occurred mostly on tracts where only one bid was submitted, usually by a neighbour firm, and where the value of that bid was low, less than \$1 million. Bids over \$5 million dollars were almost always accepted. Hendricks, Porter, and Spady concluded that 'the purpose of the government's rejection policy was to reduce the incentive that firms might have had to bid the pre-announced minimum price on tracts that, on the basis of public information, were regarded as low value tracts.'

In summary, a drainage auction can be viewed as a first-price sealed-bid auction having one informed potential buyer, an arbitrary number of uninformed potential buyers, and a random reserve price that is set by a possibly informed seller. Hendricks, Porter, and Wilson (1992) have studied this type of auction and characterized equilibrium behaviour when the joint distribution of (V, X, R) satisfies plausible regularity conditions. Two sets of empirically testable predictions follow from their analysis.

One set concerns the profits earned by neighbour and non-neighbour firms. Equilibrium implies that the expected profits are zero for non-neighbour firms, and strictly positive for neighbour firms. The difference is a measure of the value of the neighbour firm's informational advantage. Since a significant fraction of the tracts were dry, the ability to identify which tracts possess oil and gas should have been worth several millions of dollars. Hendricks and Porter (1988, 1992) found that this was indeed the case. For the period in which their measure of expected returns did not contain too much measurement error, net profits of neighbour firms were roughly \$5 million per tract, while the net profits of non-neighbour firms were not significantly different from zero. Moreover, earnings of non-neighbour firms were significantly negative on tracts where no neighbour firms bid. This reflects the adverse selection problem that non-neighbour firms experienced in bidding against the better-informed neighbour firms.

The above result can be criticized as a weak test of the game-theoretic model. The only prediction that may be difficult to obtain from alternative (non-strategic) models is that non-neighbour firms should participate with positive probability and earn zero expected profits. Non-neighbour firms have to recognize that their participation is necessary to keep the neighbour firm from bidding too low on valuable tracts. At the same time, they cannot bid naïvely but must participate, so that the optimal response of the neighbour firm allows them to break even.

The second set of predictions concerns the distributions of neighbour and non-neighbour bids. The key regularity condition which Hendricks, Porter, and Wilson (1992) impose upon the joint distribution of (V, X, R) is that these random variables be *affiliated*, a concept first introduced in the bidding literature by Milgrom and Weber (1982). Roughly speaking, affiliation requires (V, X, R) to be non-negatively correlated on any rectangle in $\Re^{P+2}$, a plausible assumption in the case of oil and gas leases. Affiliation implies that the expected value of V, conditional upon X and R, is non-decreasing in the realizations of X and R. Affiliation also implies that the distribution of R, conditional upon X, satisfies the monotone likelihood ratio property with respect to be realizations of X.

Given the affiliation assumption, Hendricks, Porter, and Wilson have demonstrated that equilibrium imposes a set of restrictions upon the distribution of B_I, the bid of the informed potential buyer, and the distribution of B_U, the maximum bid submitted by uninformed potential buyers. Let $G_I(b; z)$ denote the distribution of B_I conditional upon $Z = z, G_U(b; z)$ denote the distribution of B_U conditional upon $Z = z$, and $\bar{r}(z)$ denote the upper bounds of the support of R conditional upon $Z = z$. The relations between G_I and G_U for each realization z of Z are as follows:

$$\frac{g_U(b;z)}{G_U(b;z)} \leqq \frac{g_I(b;z)}{G_I(b;z)} \text{ for } G_I(b;z), G_U(b;z) > 0; \tag{R1}$$

$$G_I(b;z) = G_U(b;z) \text{ for } b \geqq \bar{r}(z); \tag{R2}$$

$$\text{For some } b(z) \ni G_I(\underline{b}(z);z) > G_I(0;z) \text{ and } G_U(\underline{b}(z);z) = G_U(0;z) \tag{R3}$$

where for simplicity it is assumed that G_I and G_U are differentiable.

Relation (R1) states that a proportional rate of increase in the distribution of the maximum uninformed bid is never greater than a proportional rate of increase in the distribution of the informed bid. The inequality is strict in the support of the reserve price. Relation (R2) states that the distributions are identical for bids above the support of the reserve price. Relation (R3) states that either the informed potential buyer submits his lowest bid $\underline{b}(z)$ with positive probability and G_U is continuous, or there exists an interval around the lowest informed bid $\underline{b}(z)$ where the uninformed potential buyers never bid.

The above restrictions imply that uninformed potential buyers are less likely to participate than the informed potential buyer, but when they do participate, they bid high rather than low. The probability of uninformed potential buyers' participating and the range of bids at which they participate depends upon the distribution of the reserve price. In drainage auctions, the probability of rejection is quite high at bids below \$1 million. Consequently, the informed potential buyer is forced to bid close to his valuation when it lies in this range. This makes low bids unprofitable for uninformed potential buyers, since they would tend to win only when the value of the tract is less than the amount bid. By contrast, bids over \$5 million are rarely rejected, so uninformed potential buyers need to participate in this range to keep the informed potential buyers' profit margin at reasonable levels. Thus, roughly speaking, the implications of (R1) to (R3) for the sample of drainage auctions are: first, a lower collective participation rate for non-neighbour firms than for neighbour firms; second, few non-neighbour bids below approximately \$1 million; and third, distributional equivalence at bids above \$4 or \$5 million.

Hendricks, Porter, and Wilson have used the empirical distributions of B_I and B_U to test these predictions and found remarkably strong support for the theory. The tests were based upon partitioning the set of all positive neighbour and highest non-neighbour bids into eight equally sized subsets according to their rank, and comparing $\Delta\hat{G}_I/\hat{G}_I$ and $\Delta\hat{G}_U/\hat{G}_U$ for each subset, where the '^'s on the G_is ($i = I, U$) denote the empirical distribution function of the bids for the sample. Ignoring any variation in Z across tracts, Hendricks, Porter, and Wilson have used the empirical distributions of B_I and B_U to test the above predictions. The tests were based upon non-parametric tests that apply the Wilcoxon rank sum statistic to establish statistical significance of one-sided departures from equivalence in the lower bid range, and equivalence in the upper bid range.

The results provide support for the theory, but they need to be interpreted with caution. First, the sample actually consists only of tracts which received at least one bid. Tracts that received no bids are not included in the sample, a situation which occurred whenever non-neighbour firms choose (randomly) not to participate and the neighbour firm's estimate of the tract's value given its information was less than \$25 per acre, the pre-announced minimum reserve price. Let r denote the lower bound of the support of the reserve price, and let 0 denote no bid. Recall that randomization by non-neighbour firms means that B_U is independent of B_I conditional upon Z. Thus, the probability of observing a neighbour firm bidding $b \geqq r$ or less is given by

$$\bar{G}_I(b; z) = \frac{[G_I(b; z) - G_I(\underline{r}; z)G_U(\underline{r}; z)]}{[1 - G_I(\underline{r}; z)G_U(\underline{r}; z)]}.$$

Similarly, the probability of observing a maximum non-neighbour bid of b or less is given by

$$\bar{G}_U(b; z) = \frac{[G_U(b; z) - G_I(\underline{r}; z)G_U(\underline{r}; z)]}{[1 - G_I(\underline{r}; z)G_U(\underline{r}; z)]}.$$

Thus, ignoring the variation in Z, the ratios $\Delta\hat{G}_I/\hat{G}_I$ and $\Delta\hat{G}_U/\hat{G}_U$ are actually estimates of $\bar{g}_I/\bar{G}_I$ and $\bar{g}_U/\bar{G}_U$. This creates a problem for testing the equivalence of g_I/G_I and g_U/G_U, since auxiliary assumptions concerning the probability of the no-bids event need to be made. Fortunately, it is not a problem for testing an implication of (R1) and (R2); viz., that G_I stochastically dominates (in the first-order sense) G_U. The reason is that $G_I = G_U$ if and only if $\bar{G}_I = \bar{G}_U$.

The second problem is more serious. Although (R1) to (R3) hold for every value of Z, tract heterogeneity makes statistical inference difficult, since the asymptotic distribution of the test statistics may not be a standard normal distribution. Moreover, much of the variation in Z is unlikely to be observed by the researcher. This implies that B_I and B_U are not necessarily independently distributed in the sample. Indeed, Hendricks, Porter, and Wilson found this to be the case in the sample of drainage tracts. The lack of independence further complicates the computation of the relevant asymptotic distributions. Hence, the authors claim that the test results are suggestive, but not definitive.

Having established that the data are not inconsistent with the predictions of the theoretical model, the next step would be to identify the probability law for (V, R, X) by imposing the equilibrium relations on the data. Is this feasible for drainage auctions? The complicating feature of the model is the possible correlation between the random reserve price and the information of the informed potential buyer. If the government had used a fixed reserve price (or, alternatively, if R and X are independent), then the task would be considerably easier. The reason is that, regardless of the dimension of X, the information of the informed potential buyer can be summarized by his conditional expectation, $H = E[V|X]$, which is a real-valued random variable. The bidding strategy of the informed potential buyer is simply a function that maps realizations of H into bids. One could then conduct an analysis (parametric or non-parametric) similar in spirit to that outlined in the next sections to determine the distribution of H conditional upon Z. But, from a practical perspective, the unobservability of important elements of Z suggests that such an analysis is unlikely to succeed.

Moreover, in drainage auctions, knowledge of the structural parameters of the joint distribution of (V, R, X) is unimportant for mechanism design (see Hendricks, Porter, and Tan 1993), as opposed to knowing whether the predictions of the theory match the data. If the economic environment can be characterized by asymmetric information, a random reserve price, and approximately risk-neutral, Bayesian-Nash behaviour, then there is sufficient information to assess the institution. Of course, structural estimation would be useful to the extent that exact predictions of the theory can be tested.

III. STRUCTURAL MODELS I: ENGLISH AUCTIONS

In this section, we study structural estimation of English auctions within the independent private values paradigm, and discuss applications of the procedure by Paarsch (1989, 1991, 1993). The simplifying feature of this bidding environment is that no strategic issues exist to complicate the analysis. Each potential buyer has a dominant strategy, to tell the truth. This allows us to focus upon the purely statistical issues involved in structural estimation of bidding models.

Consider the following bidding environment. An indivisible object is to be sold at auction with an announced minimum (reserve) price v_0 set by the seller. There are N potential buyers. The ith potential buyer has a valuation v_i for the object, which is known to him but not to his $(N - 1)$ opponents. Heterogeneity across potential buyers in valuations is ascribed to independent draws from a common probability density function $f(v)$, with cumulative distribution function $F(v)$, having support upon the interval $[\underline{v}, \bar{v}]$. We shall assume that v_0 exceeds $\underline{v}$. The number of potential buyers N and the distribution function $F(v)$ are assumed to be common knowledge. Potential buyers are assumed to be risk neutral with respect to winning the object.

This paradigm is known as the independent private values paradigm. It is the simplest possible bidding environment within which to test auction theory or to use auction theory to estimate the joint distribution function of the N valuations $(V_1, V_2, \ldots, V_N)$. The reason is due to two important assumptions: the independence of valuations and the absence of any observable characteristics distinguishing buyers. Together, these two assumptions imply that the joint distribution function factors into N identical marginal distributions. Thus, estimating F is equivalent to recovering the entire joint distribution for the valuations of potential buyers. Furthermore, every bid (and, in particular, the winning bid) provides useful information concerning F. This is important since, in many instances, only winning bids are observed by the researcher. Throughout this section, we shall also assume that the researcher observes N.

The simplest auction from which to estimate F is an English auction. Suppose the seller sets the reserve price at v_0 and then lets it rise more or less continuously as long as at least two potential buyers are willing to pay the announced price. Each potential buyer indicates his willingness to pay by some action that is observable not only to the seller but also to his opponents. Within this setting, the optimal strategy for each potential buyer is to participate as long as the announced sale price does not exceed his valuation. Thus, the winner is the potential buyer with the highest valuation. If he is the only potential buyer willing to participate at v_0, then the price he pays is v_0. Otherwise, he pays a price equal to the second-highest valuation. (Here we ignore any discreteness in the bidding process.)

More formally, letting $V_{(i:N)}$ denote the ith highest order statistic for a sample of size N from the distribution of V and denoting the winning bid by W, the above bidding behaviour implies that W equals $V_{(2:N)}$ whenever W exceeds v_0. The probability density function for W in this case is given by

$$h_E(w) = N(N-1)[1 - F(w)]F(w)^{N-2}f(w).$$

If W equals v_0, then all that one can infer about F is that $(N - 1)$ draws are less than v_0 and one exceeds v_0. To calculate the probability of this event, introduce the indicator variable

$$P_i = \begin{cases} 1 & \text{if } V_i \geqq v_0 \\ 0 & \text{otherwise.} \end{cases}$$

The number of participants at an auction is

$$n = \sum_{i=1}^{N} P_i,$$

where n is distributed binomially with parameters N and $Pr[P_i = 1] = [1 - F(v_0)]$. Thus, the probability of only one potential buyer participating, a winning bid of v_0, is

$$h_E(v_0) = NF(v_0)^{N-1}[1 - F(v_0)],$$

while the probability of the object going unsold, $n = 0$ or $W = 0$, is

$$h_E(0) = F(v_0)^N.$$

Notice that the density function for the winning bid has discrete mass points as well as a continuous density even though the density of v is strictly continuous.

Since the reserve price v_0 exceeds V, the number of actual bidders (participants) at an auction n is endogenous, and typically less than the number of potential buyers N. Only those potential buyers with valuations exceeding the reserve price v_0 participate.

In general, auctions are not identical, and can differ according to certain observable characteristics. Letting t index a random sample of T auctions, we adopt the parametric formulation

$$F_t(v) = F(v; \theta, Z_t),$$

where Z_t is a vector of characteristics that are observable not only to all of the potential buyers, but also to the researcher, and where θ is a $(p \times 1)$ vector of unknown parameters that characterize the shape of $F(\cdot)$. Thus, F represents the distribution of the unobserved heterogeneity embodied in private valuations conditional upon the characteristics of the object being sold. With independence across auctions as well as variation in the number of potential buyers N_t, the likelihood function takes the form

$$L(\theta) = \prod_{t=1}^{T} \frac{h_E(v_0; \theta, Z_t, N_t)^{D_t} h_E(w_t; \theta, Z_t, N_t)^{1-D_t}}{[1 - h_E(0; \theta, Z_t, N_t)]},$$

where

$$D_t = \begin{cases} 1 & \text{if } n_t = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Here the indicator variable D_t is a switch that describes the level of competition. The contributions for one participant and for more than one participant are scaled by one minus the contribution for no participants to reflect the fact that most samples are truncated samples: data concerning auctions that no one attends are usually unavailable. An estimate of the unknown value of θ can be calculated by maximizing $L(\theta)$ or log $L(\theta)$ with respect to θ.

The behavioural hypotheses of the model are that potential buyers bid independently and that losers tell the truth. Each potential buyer drops out of the bidding when the price reaches his valuation, thereby revealing his valuation. The optimality of this strategy does not depend upon the strategies chosen by opponents. Hence, the equilibrium is a dominant strategy. Unfortunately, the absence of strategic play makes it difficult to test the theory, assuming F is unknown. Any assumption concerning F is an assumption concerning the distribution of bids (or the winning bid), since the bid function is essentially the identity function.

Paarsch (1989, 1991) has applied this model to data concerning timber sales in the province of British Columbia, Canada. In British Columbia, the minister of forests sets aside a portion of each year's allowable cut to sell to eligible loggers and sawmillers through a series of public auctions held under the Small Business Forest Enterprise Program (SBFEP). To be eligible, a logger or mill owner must be registered (at a cost of \$100 per year) and must not hold more than two SBFEP contracts at the time of the sale. Paarsch considered only category 1 participants in the SBFEP, those who were loggers, since only a few auctions for sawmillers (category 2 sales) were held in any district. Paarsch used the number of eligible category 1 SBFEP registrants in a forest district as a measure of N for auctions held in that district, since registrants rarely participated at auctions outside of their district. The number of eligible category 1 SBFEP registrants in a forest district is available to the public at the district office and varies across sales.

The object being sold is the right to harvest a stand of timber. Prior to the auction, the Ministry of Forests makes available a report (called the 'timber cruise') which provides information concerning the terrain and the accessibility of the stand as well as the quality and volume of timber on the stand. In addition, potential buyers can and do inspect the stand themselves. The Ministry of Forests assigns a minimum (reserve) price (known as the 'upset' price) per cubic metre, v_0^k, for each species k harvested. A bid is then a single number which the buyer agrees to pay for each cubic metre of timber harvested in addition to the minimum price. Thus, if awarded the cutting rights at price s, the buyer pays the Ministry of Forests $s + v_0^k$ for each cubic metre of species k harvested. These prices are known as 'stumpage rates.'

Paarsch assumed that each potential buyer's valuation of a stand is proportional to the volume of standing timber (or some weighted average of the volumes of the different species) less a fixed cost. The proportionality factor is the sum of two observable components, prices for logs and costs of transport, and an unobservable component which Paarsch interprets as harvesting costs. Prices for logs and costs of transport as well as timber volumes, and the number of potential buyers are the sources of observed heterogeneity. Their variation across auctions is used to identify the parameters of the net return function. Harvest costs are assumed to be independently and identically distributed across potential buyers and timber sales.

Paarsch considered two specifications for the probability law, the Pareto and the Weibull. Estimates of their parameters, together with those of the return function, are obtained using the method of maximum likelihood as described above.

Paarsch (1993) has also derived an estimate of the optimal auction.[3] Within the independent private values paradigm, the optimal auction involves selecting an optimal reserve price r. This solves the following equation:

$$r = v_0 + \frac{[1 - F(r)]}{f(r)}.$$

Assuming that the observed v_0 is the government's true valuation for the timber for sale, Paarsch estimates the optimal reserve price to be about \$10 per cubic metre, instead of \$2.39, the 'average' reserve price in his sample.

IV. STRUCTURAL MODELS II: FIRST-PRICE SEALED-BID AUCTIONS

In this section, we survey procedures for structural estimation of first-price sealed-bid auctions within the independent private values paradigm. We first discuss the derivation of the equilibrium bid functions, and how one can use the equilibrium relations to identify the underlying probability law from bid data. We then discuss three estimation strategies that have been proposed in the literature: the method of maximum likelihood proposed by Donald and Paarsch (1991); the method of simulated non-linear least squares proposed by Laffont, Ossard, and Vuong (1991); and finally, a non-parametric method proposed by Guerre et al. (in progress) and applied by Elyakime et al. (1994).

Consider once again the independent private values paradigm, but where the sale mechanism is a first-price sealed-bid auction. At these auctions, each potential buyer is asked to submit his bid privately to the seller. The object is then sold to the highest bidder at a price equal to his bid, provided this bid is at least as high as the reserve price. If more than one potential buyer makes the same high bid, then the object is allocated according to some rationing rule which is independent of the reserve price. In this setting, a (pure) strategy for the ith potential buyer is a real-valued function $\hat{\sigma}_i$ of the realization for V_i. Given a strategy profile $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_N)$, the expected pay-off to the ith potential buyer with valuation v_i from bidding $s_i = \hat{\sigma}_i(v_i)$ is

$$(v_i - s_i) \prod_{j \neq i} F(\hat{\sigma}_j^{-1}(s_i)), \tag{1}$$

where $\hat{\sigma}_j^{-1}$ denotes the inverse (possibly a set-valued correspondence) of potential buyer j's strategy. The first term in parentheses of (1) represents potential buyer i's profit margin, while the second term of (1) represents the probability of potential

3 See Riley and Samuelson (1981) for specifics concerning the optimal auction within this paradigm.

buyer i's winning the auction given bid s_i. Note that the latter depends upon the strategies of his opponents and the distribution function of their valuations. Consequently, potential buyer i's optimal strategy depends upon his beliefs concerning the bidding rules of his opponents as well as the distribution of their valuations.

To solve the game, we use a solution concept known as Bayesian-Nash equilibrium. This concept requires each potential buyer's conjectures concerning the strategies played by his opponents to be correct, and that his strategy is a best response to these conjectures. More precisely, a Bayesian-Nash equilibrium is a strategy profile σ such that for each $i = 1, \ldots, N$, σ_i is a best response to $\sigma_{-i} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots, \sigma_N)$ for every realization of V_i. In what follows, we shall focus upon equilibria at which all potential buyers use the same strategy, and that strategy is a strictly increasing, differentiable function of V_i.

The first-order condition for optimality of s_i is

$$(v_i - s_i)\left[\sum_{j \neq i} f\left(\hat{\sigma}_j^{-1}(s_i)\right) \frac{d\hat{\sigma}_j^{-1}(s_i)}{ds_i} \prod_{k \neq j} F\left(\hat{\sigma}_k^{-1}(s_i)\right)\right] - \prod_{j \neq i} F\left(\hat{\sigma}_j^{-1}(s_i)\right) = 0. \quad (2)$$

If (2) is assumed to hold for all values of V_i and the condition of symmetry is imposed, then (2) becomes a differential equation in $\hat{\sigma}$

$$\hat{\sigma}'(v) + \frac{(N-1)f(v)}{F(v)}\hat{\sigma}(v) = \frac{(N-1)vf(v)}{F(v)} \quad (3)$$

where $\hat{\sigma}'(v)$ is the derivative of $\hat{\sigma}(v)$ with respect to v. Using the boundary condition $\hat{\sigma}(v_0) = v_0$, this differential equation can be solved for valuations above the reserve price to obtain the equilibrium bid function

$$\sigma(v) = v - \frac{\int_{v_0}^{v} F(u)^{N-1} du}{F(v)^{N-1}} \quad v \geqq v_0. \quad (4)$$

Equation (4) states that potential buyers with valuations above the reserve price bid their valuation less a 'shading' factor, the magnitude of which reflects the amount of competition N, where a potential buyer is in the distribution $F(v)$, and the reserve price v_0. Note that σ is strictly increasing in v, as required by the initial hypothesis. For those with valuations less than v_0, no bid (which we denote by zero) is submitted. This participation condition is identical to that at English auctions. It implies that the number of participants at the auction n is once again endogenous.

The winner of the auction will be the potential buyer with the highest valuation $V_{(1:N)}$. The winning bid function is monotonic in $V_{(1:N)}$, and its distribution is related to that of the largest order-statistic for a sample of size N from the distribution of V. This distribution is different from that for the winning bid at an English auction. However, under the assumption of risk neutrality, the revenue equivalence proposition implies that the expected value of the winning bid at a first-price sealed-bid auction equals that at an English auction; see Riley and Samuelson (1981).

Exploiting the fact that (4) is a monotonic function of V (the higher is a potential buyer's valuation, the more he will bid) provides one strategy for interpreting field data; see, for example, Paarsch (1989, 1991, 1992, 1993). Because the bidding rules are functions of the random variable V, the bids are also random variables and their densities are related to $f(v)$. Thus,

$$G_S(s) = F(\sigma^{-1}(s)) \text{ and } g_S(s) = \frac{dG_S(s)}{ds} = \frac{f\left(\sigma^{-1}(s)\right)}{\sigma'\left(\sigma^{-1}(s)\right)}. \tag{5}$$

Note that the support of the bid distribution function is on the interval $[v_0, \bar{s}]$, where $\bar{s} = \sigma(\bar{v})$.

An interesting question to ask is whether another latent distribution exists, other than F, which would be consistent with the observed data G_S. This is a non-trivial identification problem in the case of first-price sealed-bid auctions because, even though σ is invertible in V, it depends upon F.

For the case of first-price sealed-bid auctions within the independent private values paradigm, Donald and Paarsch (1992) have elaborated upon a set of necessary and sufficient conditions which permit identification by working directly off (4). These turn out to be quite innocuous, and to be met by virtually all parametric distributions which a researcher might consider using in practice.

Guerre et al. (in progress) (as discussed in Elyakime et al. 1994) work directly with the differential equation (3). They substitute the relations given in (5) into the differential equation (3), and solve for v

$$v = s + \left[\frac{(N-1)g_S(s)}{G_S(s)}\right]^{-1} \equiv \xi(s). \tag{6}$$

Guerre et al. show that the necessary and sufficient conditions for identification are that $\xi(s)$ is strictly increasing in s, and that $\lim_{s \to v_0} \xi(s) = v_0$. In words, if $g_S(s)/G_S(s)$ is well behaved, then there exists a unique F that makes G_S the distribution for any one of the equilibrium bids. Note that $\xi(s)$ is simply the inverse function $\sigma^{-1}(v)$, and that this approach to identification does not require an explicit solution to (3).

1. Maximum likelihood

To keep the analysis similar to that of English auctions, we shall assume that the researcher uses only the winning bid.[4] The winning bid is a function of the $\{V_i\}_{i=1}^N$. Hence, its density is related to $f(v)$. The density of the winning bid at a first-price sealed-bid auction ($V_{(1:N)}$), denoted $h_S(w)$, is

$$\begin{aligned} h_S(w) &= \frac{NF\left(\sigma^{-1}(w)\right)^{N-1} f\left(\sigma^{-1}(w)\right)}{\sigma'\left(\sigma^{-1}(w)\right)} \\ &= \frac{NF\left(\sigma^{-1}(w)\right)^{N}}{(N-1)\left(\sigma^{-1}(w) - w\right)}. \end{aligned}$$

4 In most instances, every bid above the reserve price is observed, but the modifications to the analysis are straightforward.

The last equality in (7) is obtained by substituting in the derivative of σ. When the object goes unsold, one knows only that $V_{(1:N)}$ is less than v_0, which one can calculate using the method presented in section III. Hence,

$$Pr[n = 0] = F(v_0)^N.$$

Maximum likelihood estimation requires that the researcher adopt a parametric specification for the probability law for each auction $t = 1, \dots, T$. As in section III, let

$$F_t(v) = F(v; \theta, Z_t),$$

where Z_t is a vector of observed covariates. Estimation by the method of maximum likelihood is not as straightforward as in the case of English auctions. The difficulty arises from the fact that the equilibrium bid function at first-price sealed-bid auctions depends upon $F(v; \theta, Z_t)$. This in turn implies that $\bar{s}(\theta, Z_t, N_t)$, the upper bound of the support of $h_S(w; \theta, Z_t, N_t)$, depends upon the parameters of interest, which violates the standard regularity conditions required to demonstrate the consistency of the maximum likelihood estimator and to derive its asymptotic distribution.

Donald and Paarsch (1991) have proposed a maximum likelihood estimator. The estimator requires maximizing the likelihood of the observed sample subject to the constraints that the bidding outcomes be consistent with the implied upper bounds of the bid distributions. More formally, the maximum likelihood estimator is defined as the solution to the following non-linear programming problem:

$$\max_{\langle\theta\rangle} \sum_{t=1}^{T} \left[\log h_S(w_t; \theta, Z_t, N_t) - \log\left[1 - F(v_{0t}; \theta, Z_t)^{N_t}\right] \right]$$

subject to

$$w_1 \leqq \bar{s}(\theta, Z_1, N_1)$$

$$w_2 \leqq \bar{s}(\theta, Z_2, N_2)$$

$$\vdots$$

$$w_T \leqq \bar{s}(\theta, Z_T, N_T).$$

The second term within the large brackets of the sum defining the objective function represents a correction for the fact that auction samples are typically truncated samples: only those auctions at which at least one potential buyer competes enter data sets.

Donald and Paarsch have demonstrated that the maximum likelihood estimator is not simply consistent, but is super-consistent: the estimator converges to the true parameter vector at rate T instead of the typical rate $\sqrt{T}$, a fact arising from the use of extreme value statistics instead of averages to define the estimator. This quick rate of convergence is very important in applications to auctions where sample sizes of fifty are common, and those of 200 would be considered large.

The solution that defines the maximum likelihood estimator typically obtains at the intersection of the constraints, so the properties of the perturbed optimum of the logarithm of the likelihood function are determined solely by the constraints. Consequently, the distribution theory for the estimator is quite complicated. Indeed, with discrete covariates, Donald and Paarsch show that the limiting distribution of a linear transformation of the estimator falls within a $(p \times 1)$ dimensioned vector of independently and identically distributed exponential random variables. Although Donald and Paarsch have demonstrated the consistency of the maximum likelihood estimator when the covariates are continuous, no distributional results have been established yet for this case.

Paarsch (1989, 1991) has applied the above structural framework to data concerning timber sales in British Columbia, Canada. Several forest districts used first-price sealed-bid auctions rather than English auctions to sell the right to harvest timber. Moreover, the choice of mechanism appears random – uncorrelated with economic factors like prices for logs, volumes of standing timber, or 'upset rates' – suggesting that the underlying probability law for the valuations of potential buyers may have been the same across the two types of auctions. If so, the estimate of F obtained from the two data sets should be the same. The results have not supported this hypothesis. In particular, the participation behaviour of potential buyers seemed to differ across the two mechanisms, since the fraction of auctions at which the winning bid was the reserve (upset) price was significantly higher at English auctions than at first-price sealed-bid auctions. Paarsch has found some evidence suggesting that potential buyers were not risk neutral with respect to winning the auction.

This empirical work illustrates the potential of structural models. Theory predicts that average revenues at English and first-price sealed-bid auctions should be the same if valuations are independently and identically distributed and potential buyers bid competitively. In the case of timber sales in British Columbia, this implication does not appear to hold, even after one accounts for observable heterogeneity. The reasons for this may be mis-specification of the structural elements or inappropriate behaviour. In either case, structural methods are central to evaluating the significance of differences in outcomes and to explaining them.

2. Simulated non-linear least squares

The main drawback to applying the method of maximum likelihood is numerical complexity. To calculate the contribution of (w_t, Z_t, N_t) to the logarithm of the likelihood function, one needs to compute the function $\sigma^{-1}(w_t; \theta, Z_t, N_t)$. But except in special cases, this function (and, for that matter, $\bar{s}(\theta, Z_t, N_t)$) do not have closed-form solutions and typically will need to be evaluated numerically at every observation and every trial value of θ. The computational burden of this exercise can be high and can effectively limit the class of distributions which the researcher can entertain.

Laffont, Ossard, and Vuong (1991) have proposed an alternative estimation strategy which can circumvent some of these computational difficulties in certain circumstances. The strategy of Laffont, Ossard, and Vuong is called the method of 'simulated non-linear least squares,' and it is closely related to the method of 'simulated moments' proposed by McFadden (1989) and Pakes and Pollard (1989).

Laffont, Ossard, and Vuong have noted that within the independent private values paradigm, the equilibrium bid function for the tth auction can be written as the following conditional expectation:

$$\sigma_t(v) = \mathrm{E}[\max\{v_{0t}, V_{(2:N_t)}\} | V_{(1:N_t)} = v].$$

Since the expectation of the winning bid at auction t corresponds to the expectation of second-highest private value,[5] this implies that

$$\mathrm{E}[W_t] = \mu(\theta, Z_t, N_t) = \mathrm{E}[\max\{v_{0t}, V_{(2:N_t)}\}].$$

Integrating over all possible realizations of the second-highest valuation then yields

$$\mu(\theta, Z_t, N_t) = \int_0^\infty \ldots \int_0^\infty \max\{v_{0t}, V_{(2:N_t)}\} f(v_1; \theta, Z_t, N_t) \ldots f(v_{N_t}; \theta, Z_t, N_t) dv_1 \ldots dv_{N_t}. \quad (8)$$

Equation (8) is used as the basis of the proposed simulated non-linear least squares estimator.

The idea is straightforward. For each of $t = 1, \ldots, T$ observations, J independent simulation samples of size N_t are drawn; (8) is then approximated by the sample mean of this function for each of the simulated samples. Following Gourieroux and Monfort (1990), Laffont, Ossard, and Vuong use the concept of the 'importance function' (common in Bayesian Monte Carlo integration) to admit the possibility that the parameter vector to be estimated θ is part of the function to be simulated $\mu(\theta, Z, N)$. Letting $\psi_t(\cdot)$ denote the importance function – which the researcher must specify ex ante – Laffont, Ossard, and Vuong estimate (8) by

$$\bar{m}(\theta, Z_t, N_t) = \frac{1}{J} \sum_{j=1}^{J} m_j(\theta, Z_t, N_t)$$

where

$$m_j(\theta, Z_t, N_t) = \max\{v_{0t}, v^j_{(2:N_t)t}\} \frac{f_t(v^j_{1t}; \theta) \ldots f_t(v^j_{N_t t}; \theta)}{\psi_t(v^j_{1t}) \ldots \psi_t(v^j_{N_t t})}.$$

5 When potential buyers are risk averse, this equality will be violated. Also, within other paradigms this equality will not, in general, hold.

Here, each of the v_{it}^{j}s represents an independent draw for a random variable having the probability density function $\psi_t(\cdot), i = 1, \ldots, N_t; j = 1, \ldots, J$; and $t = 1, \ldots, T$.

Laffont, Ossard, and Vuong have noted that using $\bar{m}(\theta, Z_t, N_t)$ as $\mu(\theta, Z_t, N_t)$ when minimizing the objective function

$$\sum_{t=1}^{T}(w_t - \bar{m}(\theta, Z_t, N_t))^2$$

with respect to θ will produce an inconsistent estimator of θ for any finite number of simulations J because $\bar{m}(\theta, Z_t, N_t)$ estimates $\mu(\theta, Z_t, N_t)$ with error. They have derived the simulated non-linear least squares estimator of θ, which is consistent for a finite fixed J simulations, by minimizing the following objective function:

$$Q(\theta) = \sum_{t=1}^{T}\left[\left(w_t - \bar{m}(\theta, Z_t, N_t)\right)^2 - \frac{1}{J(J-1)}\sum_{j=1}^{J}\left(m_j(\theta, Z_t, N_t) - \bar{m}(\theta, Z_t, N_t)\right)^2\right]$$

with respect to θ. The bias introduced by the pre-estimation error in $\bar{m}(\theta, Z_t, N_t)$ is eliminated asymptotically by introducing the second term of $Q(\theta)$, which represents an estimate of the sample variance of $\bar{m}(\theta, Z_t, N_t)$. Laffont, Ossard, and Vuong have shown that the simulated non-linear least squares estimator is distributed asymptotically normal, and they derive an estimator of the variance-covariance matrix for the simulated non-linear least squares estimator.

Laffont, Ossard, and Vuong have applied their method of estimation to data from daily sales of eggplants at a market in Marmande, France, during the summer of 1990. Each day several lots of eggplants of variable size – 15 to 350 kilograms – are sold sequentially using a descending auction. The seller announces a reserve price per unit (i.e., a price per kilogram) and then starts the bidding at a very high unit price. The price drops quickly until one of the potential buyers makes a bid. The first potential buyer who makes a bid before the reserve price is reached wins the lot and pays his bid. This one-shot descending (or Dutch) auction is strategically equivalent to a first-price sealed-bid auction when the valuations of potential buyers are independently and identically distributed. Laffont, Ossard, and Vuong assumed that F belongs to the log-normal family of distributions.

Laffont, Ossard, and Vuong faced two difficulties in their application which are likely to present problems for any researcher seeking to do empirical work on this type of auction. At Dutch auctions, only the winning bid is observed, since losers are not required to submit their bids. This makes it difficult for a researcher to measure the number of potential buyers. Laffont, Ossard, and Vuong inferred the presence of at least eleven buyers, since there were eleven different winners in their sample. But this does not mean that each potential buyer attended every auction, nor does it account for the possibility of potential buyers who do not win. The authors resolve this problem by treating N as a parameter to be determined by the data.

The second problem is to justify the independent private-values paradigm. The potential buyers are agents of retail sellers who serve distinct markets and place orders with their agents at specific prices before the opening of the market each day. The sources of observable heterogeneity common to all potential buyers are quantity and quality of supply and variables measuring the general state of demand. Variation in valuations across retailers is then attributed to idiosyncratic factors in each retailer's daily market. Furthermore, for each retailer, these factors are random variables which are assumed to be independently and identically distributed across market days. Thus, the sample selected consisted of one auction per day for each market day in the period from June to October 1990. Nevertheless, the assumption of independence across market days seems quite strong.

3. Non-parametric estimation

One criticism of the method of maximum likelihood or the method of simulated non-linear least squares is that an explicit distributional assumption must be made concerning $f(v)$. Recently, Elyakime et al. (1994) have implemented the non-parametric methods of Guerre et al. (in progress) to estimate F at first-price sealed-bid auctions. They exploit equation (6), which expresses the unobserved valuation of a potential buyer in terms of his bid and the potentially estimable quantity $g_S(s)/G_S(s)$. Hence, in contrast to parametric methods, this approach requires knowledge of all of the bids, not just the winning bid. The main behavioural assumptions are that each potential buyer is bidding optimally against the bidding distribution generated by his opponents' strategies, that all potential buyers are using the same strategy, and that the strategy is increasing in v.

Application of the procedure consists of two steps. The sample is the set of all pairs $\{(s_l, Z_l) : l = 1, \ldots, \sum_{t=1}^{T} n_t\}$, where $\sum_{t=1}^{T} n_t$ is the total number of bids submitted at the T auctions. Treating each observation as an independent draw, the first step consists of estimating the conditional density $g_S(s|Z)$ non-parametrically. This step produces $\hat{g}_S(s|Z)$ and $\hat{G}_S(s|Z)$ which can be used in the second step to estimate the 'hazard rate'

$$\frac{g_S(s|Z)}{G_S(s|Z)}.$$

The second step consists of plugging in the non-parametric estimates of the 'hazard rate' into (6) and evaluating $\xi(s|Z)$ at each bid s_l to generate a fitted sample $\{(v_l, Z_l) : l = 1, \ldots, \sum_{t=1}^{T} n_t\}$. This sample can be used to obtain a non-parametric estimate of the bidding strategy $\hat{\xi}^{-1}(s|Z)$ and of the conditional density of valuations $\hat{f}(v|Z)$.

This non-parametric estimation method identifies the underlying probability law for the valuations of buyers. Moreover, as Elyakime et al. have pointed out, it also provides a test of the behavioural hypothesis. The non-parametric estimates of $g_S(s)/G_S(s)$ can produce an estimate $\hat{\xi}(s|Z)$ that is not everywhere increasing. Nor does the procedure require $\hat{\xi}(v_0|Z)$ to equal v_0. These properties are necessary implications of the theory. Consequently, if the estimated bid strategy is increasing and passes through the reserve price, this is evidence consistent with the theory.

Elyakime et al. have applied this method to data concerning timber sales in Tarn, France. A cooperative of timber owners sells the right to harvesting standing timber to sawmills. The market is held twice each year. The characteristics of each lot are described in a booklet which is freely available to potential buyers prior to each sale. In addition, potential buyers can inspect the stands. Each stand is auctioned successively on the day of the sale at a first-price sealed-bid auction. The main source of observable heterogeneity which Elyakime et al. have considered is the number of actual bidders and the percentage of saw timber.

The interesting theoretical feature of these auctions is that the seller's reservation price is not announced until after the bids have been submitted. Hence, any interested buyer must submit his bid in ignorance of the reserve price. This is particularly useful from an empirical perspective, since in the absence of participation costs it implies no truncation of the bids. However, it does introduce an asymmetry into the model. From the perspective of potential buyers the seller is another potential buyer, but one who has a different pay-off function. This implies a bidding strategy for the seller that is different from strategies of the potential buyers. Consequently, the optimal strategy of potential buyers cannot be obtained explicitly, and the researcher must work with a differential equation like (3) as the basis of the empirical model.

4. Remarks

Parametric methods require the researcher to assume that F belongs to a particular family of distributions. Hence, even if one takes as given that the valuations of potential buyers are independently and identically distributed random variables, the hypothesis being tested is always a joint hypothesis involving both functional form and behavioural assumptions. This is not uncommon in economics. Consider, for example, much of the research devoted to testing consumer and producer theory.

The maximum likelihood estimator is feasible in bidding environments where the differential equation(s) characterizing equilibrium behaviour can be solved explicitly and the solution has a closed-form. The simulated non-linear least squares estimator requires an explicit solution to the differential equation but does not impose any further restrictions. Such solutions generally exist when potential buyers are risk neutral and valuations are independently and identically distributed random variables.

Paarsch (1994) has compared the performance of the maximum likelihood and simulated non-linear least squares estimators. He found that importance sampling should be avoided, since the simulated non-linear least squares estimator is quite sensitive to both the choice of importance sampling function and the parameters embedded in that function.

An impressive fact that emerged from Paarsch's work is that the simulated non-linear least squares estimator was relatively accurate, even when the simulation samples were small. Moreover, in the examples he examined, the use of simulation did not increase the variance of the estimator very much. There were some problems with bias, but these appeared to be relatively minor in Paarsch's example. What was disturbing was that the bias fell very slowly with sample size. Paarsch also found that the maximum likelihood estimator was biased, but that this bias was reduced quickly with increases in the sample size.

In general, although the maximum likelihood estimator requires much more structure than the simulated non-linear least squares estimator, it has one redeeming feature: it converges at rate T, while the simulated non-linear least squares estimator only converges at rate $\sqrt{T}$. In practical terms, this means that an empirical researcher can obtain as much accuracy with a sample size of fifty when using the method of maximum likelihood as he could with a sample size of 2,500 when using the method of simulated non-linear least squares. This is of particular relevance when field data from actual auctions are investigated, because sample sizes of fifty are common, and those of 200 would be considered large.

The non-parametric estimator is computationally quite simple. It requires no prior restrictions on the underlying probability law for the valuations, except for independence. It provides a weak test of the theory in that $\hat{\xi}(s)$ need not be strictly increasing and $\hat{\xi}(v_0)$ need not equal v_0. The main disadvantage of the method is inefficiency, because the rates of convergence for non-parametric methods are typically much slower than $\sqrt{T}$: large samples are required, particularly if there are several important covariates.

V. SUMMARY AND CONCLUSIONS

In applying a theoretical model to data, the first step is to determine whether the observed behaviour of agents is broadly consistent with the predictions of the model. For drainage auctions, the game-theoretic model of a single informed potential buyer, an arbitrary number of uninformed potential buyers, and a random reserve price yields a number of predictions which, taken as a group, provide a test of the model. As Hendricks, Porter, and Wilson note in the conclusion of their paper, it is difficult to develop an alternative behavioural model that can do as well in explaining the data.

Game-theoretic models do not always provide such a rich set of empirically testable predictions. The distinguishing feature of the drainage auctions is that there are two identifiable buyer types: neighbour and non-neighbour firms. Since the strategy of each buyer type depends upon its rival's type as well as its own type, equilibrium imposes restrictions upon relative behaviour and performance which can be examined empirically by comparing outcomes conditional upon types in a cross-section of auctions. By contrast, the bidding model for wildcat auctions, in which potential buyers have different but qualitatively similar information, fails to yield empirical predictions which allows one to differentiate it from other models because all potential buyers are essentially the same.

Structural estimation requires a much deeper level of commitment to the theoretical model. This commitment permits the researcher to identify the exact data-generating process and to exploit this information to determine the optimal mechanism. The evidence from the analyses of Paarsch (1989, 1991, 1992, 1993), Laffont, Ossard, and Vuong (1991), and Elyakime et al. (1994) has been encouraging, but more work needs to be done.

The main challenge, however, is to develop tractable empirical models of bidding environments where the valuations of potential buyers are neither independently nor identically distributed. The theory of bidding in the general affiliated values model, which nests both the independent private values and the common value models, has been worked out by Milgrom and Weber (1982). The econometric implementation of this model is still in its infancy but, if successful, would enhance significantly the domain of application. (See Laffont and Vuong 1993 for a discussion of some of the problems that are involved, particularly that of identification.) Tests of the independent private values paradigm would also be useful, especially for evaluating the structural models discussed above. Donald and Paarsch (1993b) have developed specification tests that use information from both the parametric and non-parametric approaches to test distributional assumptions and to examine for departures from independence, but more work must be done in this area.

On a more speculative note, the approach implemented by Elyakime et al. (1994) seems quite promising, at least for the independent private values paradigm. The basic idea behind their approach is to use the first-order condition for potential buyer i to express his valuation v_i in terms of his bid s_i and the sum of the empirical 'hazard rates' of his rivals' bid distributions evaluated at s_i. The researcher can then use this equation together with the sample of bids for potential buyer i to estimate the probability law for his valuation. The assumption here is that potential buyer i is bidding optimally against the observed bidding behaviour of his rivals. Note that there is no need to assume that potential buyers have the same probability law. It may then be possible to test for equilibrium behaviour by determining whether, given the estimated probability density function for potential buyer i's valuation, optimization by potential buyer i's rivals generates bid distributions that are consistent with the observed bid distributions.

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[Footnotes]

[3] **Optimal Auctions**
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